

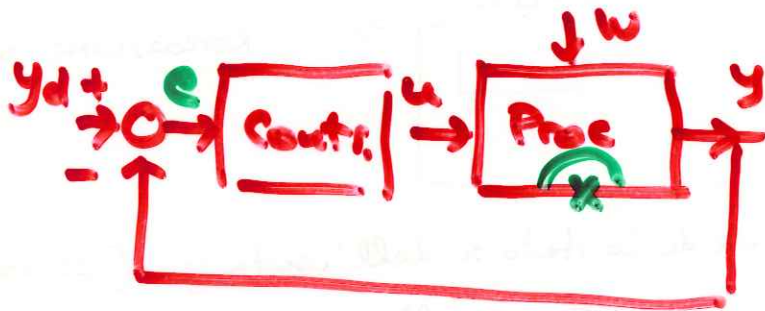
CONTROLLO basato sullo SPAZIO DI STATO

Problema

$$\dot{x} = Ax + Bu + Pw$$

$$y = Cx + Du + Qw$$

$$x \in \mathbb{R}^n \quad u \in \mathbb{R}^m \quad y \in \mathbb{R}^p \quad (P \in \mathbb{R}^m)$$



controllore istantaneo

$$u = Ke$$

" dinamico

$$\dot{\xi} = F\xi + Ge \quad \xi \in \mathbb{R}^v$$

$$u = H\xi + Le$$

TL

$$C(s) = L + H(sI - F)^{-1}G$$

- * regime permanente ② \Leftrightarrow REGOLAZIONE dell'uscita
- * regime transitorio ① (\Leftrightarrow stabilità asintotica)



$$y_d = 0$$

$$(e = -y)$$

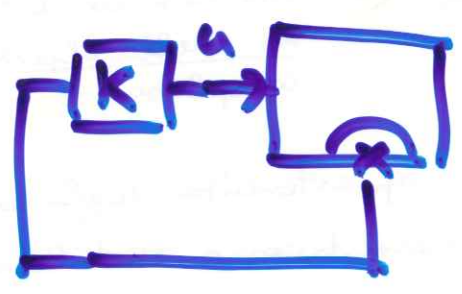
PRINCIPIO DI SEPARAZIONE

- i) REAZIONE DALLO STATO x
- ii) OSSERVATORI dello STATO x dall'uscita y

Problemi di ASSEGNAZIONE degli AUTOVALORI (con retroazione dello stato)

$$\dot{x} = Ax + Bu \quad (\cancel{y = Cx})$$

$$u = Kx \quad \text{FEEDBACK}$$



$$\dot{x} = (A + BK)x = A_{cl}x$$

- ? Quando $\exists K$: $\sigma(A_{cl}) = \sigma^*$
- ? Come ricavare K

Risposta $\exists K \Leftrightarrow (A, B)$ raggiungibile

$$\int [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B] = n$$

Dim

necessite'

Se (A, B) non è raggiungibile \rightarrow dec. Kalman
(risp. regolabile)

$\exists T$ invertibile $x \rightarrow z = Tx \quad \dot{z} = \tilde{A}z + \tilde{B}u$

$$\tilde{A} = TAT^{-1} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ 0 & \tilde{A}_{22} \end{bmatrix} \begin{matrix} \uparrow p \\ \uparrow n-p \end{matrix}$$

$$\tilde{B} = TB = \begin{bmatrix} \tilde{B}_1 \\ 0 \end{bmatrix} \begin{matrix} \uparrow p \\ \uparrow n-p \end{matrix}$$

generica $\tilde{K} = KT^{-1} = [\tilde{K}_1 \quad \tilde{K}_2]$

$$\dot{z} = (\tilde{A} + \tilde{B}\tilde{K})z = \begin{bmatrix} \tilde{A}_{11} + \tilde{B}_1\tilde{K}_1 & \tilde{A}_{12} + \tilde{B}_1\tilde{K}_2 \\ 0 & \tilde{A}_{22} \end{bmatrix} z$$

$$\tilde{p}(\lambda) = \det(\lambda I - (\tilde{A}_{11} + \tilde{B}_1\tilde{K}_1)) \cdot \det(\lambda I - \tilde{A}_{22})$$

Suff. (un ingresso) $m=1$

F. Canonici
di controllo

$$A_c = \begin{bmatrix} 0 & 1 & & 0 \\ 0 & & \ddots & \\ & & & 1 \\ -a_0 & -a_1 & \dots & -a_{n-1} \end{bmatrix} \quad b_c = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$p(\lambda) = \det(\lambda I - A) \\ = \lambda^n + a_{n-1}\lambda^{n-1} + \dots + a_1\lambda + a_0$$

$$A + b k_c^T = \begin{bmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 0 & 1 \\ -a_0 + k_1 & \dots & \dots & -a_{n-1} + k_n \end{bmatrix}$$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ c & b & k_c^T \\ n \times n & n \times 1 & 1 \times n \end{matrix}$

$$k_c = [k_1, \dots, k_n]$$

$$p^*(\lambda) = (\lambda - \lambda_1) \dots (\lambda - \lambda_n) =$$

$$\sigma_x^* \{ \lambda_1, \dots, \lambda_n \}$$

$$= \lambda^n + \alpha_{n-1}\lambda^{n-1} + \dots + \alpha_1\lambda + \alpha_0$$

$$\Rightarrow k_c = [a_0 - \alpha_0, a_1 - \alpha_1, \dots, a_{n-1} - \alpha_{n-1}]$$

per generiche (A, b) (un raggiungibili)

$$u = Kx = \xrightarrow{\quad} = k_c x_c = k_c T_c x$$

$$x_c = T_c x$$

$$T_c = \begin{bmatrix} g \\ gA \\ gA^2 \\ \vdots \\ gA^{n-1} \end{bmatrix}$$

$g =$ ultima riga di R^{-1}

$$R = [b \quad Ab \quad \dots \quad A^{n-1}b]$$

(dim. Teoria dei Sistemi)

$$\dot{x} = Ax + bu$$

$$y = gx$$

$$\begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ g \end{bmatrix} \cdot R^{-1} = \begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix} = I$$

$$\dot{y} = g\dot{x} = gAx + \cancel{gb}u$$

$$\ddot{y} = gA\dot{x} = gA^2x + \cancel{gAb}u$$

$$\frac{d^n y}{dt^{n-1}} = gA^{n-1}x + \cancel{gA^{n-2}b}u$$

$$\frac{d^n y}{dt^n} = gA^n x + \underbrace{gA^{n-1}b}_1 u$$

$$\begin{bmatrix} y \\ \vdots \\ y^{(n-1)} \end{bmatrix} = T_c x$$

$$\boxed{K = K_c T_c}$$

$$= [a_0 - \alpha_0 \quad \dots \quad a_{n-1} - \alpha_{n-1}] \begin{bmatrix} g \\ gA \\ \vdots \\ gA^{n-1} \end{bmatrix}$$

$$= [a_0 \quad \dots \quad a_{n-1}] \begin{bmatrix} g \\ gA \\ \vdots \\ gA^{n-1} \end{bmatrix} - [\alpha_0 \quad \dots \quad \alpha_{n-1}] \begin{bmatrix} \vdots \\ \dots \end{bmatrix}$$

$$= g(a_0 I + a_1 A + \dots + a_{n-1} A^{n-1}) - g(\alpha_0 I + \alpha_1 A + \dots + \alpha_{n-1} A^{n-1})$$

T. Cayley
Hamilton

$$p(\lambda) = a_0 + a_1 \lambda + \dots + a_{n-1} \lambda^{n-1} + \lambda^n$$

$$p(A) = a_0 I + a_1 A + \dots + a_{n-1} A^{n-1} + A^n \equiv 0$$

$$= -g[\alpha_0 I + \alpha_1 A + \dots + \alpha_{n-1} A^{n-1} + A^n]$$

$$\boxed{= -g \cdot p^*(A)}$$

formula di
Ackermann

Ex

$$A = \begin{bmatrix} -1 & 2 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad x = Ax + bu$$

desired poles $\sigma = \{ -1, -0.5 \pm j \frac{\sqrt{3}}{2} \}$

Butterworth



"buone" per ridurre le sovvelazioni nelle risp. indotte

reggiungibilità

$$R = [b \quad Ab \quad A^2b] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$\rho(R) = 3 = n \quad \text{!! OK}$

$$R^{-1} = \begin{bmatrix} * & * & * \\ * & * & * \\ 1 & -1 & 0 \end{bmatrix} \leftarrow g$$

$$K = -g p^*(A)$$

$$p^*(\lambda) = (\lambda + 1) \left(\lambda + 0.5 + j \frac{\sqrt{3}}{2} \right) \left(\lambda + 0.5 - j \frac{\sqrt{3}}{2} \right)$$
$$= \lambda^3 + 2\lambda^2 + 2\lambda + 1$$

$$P^*(A) = A^3 + 2A^2 + 2A + I$$

$$A^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & -2 & -1 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\dots P^*(A) = \begin{bmatrix} -2 & 2 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$K = -g P^*(A) = [-1 \ 1 \ 0] P^*(A)$$

↑ e' l'unic!

$$= [1 \ -2 \ -1] !$$

verifies!

$$A + BK \equiv \begin{bmatrix} -1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -2 & -2 \end{bmatrix}$$

$$\rightarrow \det(\lambda I - (A + BK)) = P^*(\lambda) ?$$

(S)

se il sistema non è raggiungibile

$$p(\mathcal{R}) = p < n$$

$$\dot{\tilde{x}} = \begin{bmatrix} \overbrace{\tilde{A}_{11}}^{p \times p} & \tilde{A}_{12} \\ 0 & \tilde{A}_{22} \end{bmatrix} \tilde{x} + \begin{bmatrix} \tilde{B}_1 \\ 0 \end{bmatrix} u \quad \tilde{x} = T x$$

$$u = [\tilde{k}_1, \tilde{k}_2] \tilde{x} = [\tilde{k}_1, \tilde{k}_2] T^{-1} x = k x$$

$$\dot{\tilde{x}} = \begin{bmatrix} \tilde{A}_{11} + \tilde{B}_1 \tilde{k}_1 & \tilde{A}_{12} + \tilde{B}_1 \tilde{k}_2 \\ 0 & \tilde{A}_{22} \end{bmatrix}$$

\tilde{k}_1 è unico per spostare
gli autovalori di \tilde{A}_{11} in
p locazioni desiderate

CNDS stabilizzabile $\Leftrightarrow \sigma(\tilde{A}_{22}) \subset \mathbb{C}^-$

Test di PBH (Hautus)

$$p \left[\underbrace{A - \lambda I}_{n \times n} \quad \underbrace{B}_{n \times m} \right] \Big|_{\lambda = \lambda_i} = n \Leftrightarrow \lambda_i \text{ raggiungibile}$$

$$p[A - \lambda I \quad B] = n \quad \forall \lambda = \lambda_i \text{ con } \operatorname{Re}(\lambda_i) \geq 0$$

ex

$$A = \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ \beta \end{bmatrix} \quad \beta \text{ beliebig}$$

asymptotische?

$$R = \begin{bmatrix} 1 & 1 \\ \beta & 1-2\beta \end{bmatrix} \quad \left\{ \begin{array}{l} \beta = \frac{1}{3} \Leftrightarrow \rho(R) = 1 \\ < 2 \\ \beta \neq \frac{1}{3} \Leftrightarrow \text{reguläre Matrix} \end{array} \right.$$

Stabilität?

$$\rho[A - \lambda I \mid b]_{\lambda=1} = \rho \begin{bmatrix} 0 & 0 & \vdots & 1 \\ 1 & -3 & \vdots & \beta \end{bmatrix} = 2 = n \quad \forall \beta$$

$$\rho[A - \lambda I \mid b]_{\lambda=-2} = \rho \begin{bmatrix} 3 & 0 & \vdots & 1 \\ 1 & 0 & \vdots & \beta \end{bmatrix} = \begin{cases} \beta \neq \frac{1}{3} & \rho = 2 \\ \beta = \frac{1}{3} & \rho = 1 \end{cases}$$

allgemein:

$$A + bk \quad [k_1, k_2]$$

$$\det(\lambda I - (A + bk)) = \lambda^2 + \alpha_1(k_1, k_2)\lambda + \alpha_0(k_1, k_2)$$

$$\sigma^* = \{-2, -\delta\} \rightarrow p^*(\lambda) = (\lambda + 2)(\lambda + \delta)$$